1893MNRAS..53..359T

## *Telescope Objectives for Photographic Purposes.*
### By H. Dennis Taylor.

The larger number of telescopes for mapping and surveying the heavens by means of photography have already been made, and it may be assumed that the best form of objective to satisfy the requirements has been ascertained in practice, at any rate approximately. Nevertheless, I think that the remarks which I have to offer will certainly be of interest to those numerous members of this Society who have more or less devoted their mathematical bent of mind to the kindred science of optics, with the theoretical and practical advancement of which the science of astronomy is very closely concerned.

It was wisely decided at the Paris Convention that the standard objective for celestial photography should be so corrected as to have its minimum, or most intense focus for the G ray, to which the ordinary rapid bromide plate is most sensitive. This, together with the focal length and aperture, was about all which the Convention prescribed. It did not seem to be widely enough recognised that the *form* of the objective would have a *great deal* to do with its efficiency as regards the size of field yielded by it. Dr. Steinheil, however, conscious of the importance of the *form*, made two recommendations, which were accepted by the Convention, and which ran to this effect:—The ratio between the sine of the angle of incidence on the first surface for a ray parallel to the axis, but striking the edge of the objective, and the sine of its angle of emergence at the fourth surface, must be equal to the ratio between the sines of angles of incidence at first surface and emergence at fourth surface for another ray, also parallel to the axis, but striking the objective at a point much nearer its centre. This is the condition under which the objective will give the best possible lateral image. Also, a white ray parallel to the axis, but striking the edge of the O. G, must not only have those coloured constituents which the O. G is supposed to refract to the same focus, emerge at the fourth surface in such a manner as to proceed to the same focus, but they must also emerge *at the same point* at the fourth surface; this is the condition that the images in both colours shall be of the same size.—For my part, I confess that I do not grasp the necessary relation between Dr. Steinheil's two conditions and the two advantages respectively aimed at, although I find that the first condition certainly does hold good in an objective corrected for the largest possible field. But, granting the relations to be correct, it cannot be said that Dr. Steinheil's conditions would facilitate a direct calculation of the curves, &c., necessary to fulfil the third and fourth of the several conditions following, the first three being indispensable and the last two very desirable:—

1893MNRAS..53..359T

1. The proper colour correction.
2. Freedom from spherical aberration.
3. The largest possible field of fair definition (Dr. Steinheil's first condition).
4. The images in different colours to be of the same size (Dr. Steinheil's second condition).
5. Freedom from distortion; all great circles on the celestial sphere should be represented as straight lines on the sensitive plate.

I here propose to deal more especially with the fulfilment of the third condition, although I may venture a few remarks upon the others. As to the calculation of the objective for the minimum focal length for rays of any given wave length, if nicety is required in the matter, then the only method available for a direct result is to express the focal length of the whole objective in terms of the wave length by an adaptation of Cauchy's dispersion formula, somewhat after the method of Professor Harkness (in *American Journal of Science and Arts*, vol. xviii., 1879 Sept.), differentiating the focal length, thus expressed, with respect to the wave length, and equating the differential coefficient to 0; the resulting equation will then determine the relation between $\frac{1}{\rho_1}$ for the crown lens and $\frac{1}{\rho_2}$ for the flint lens, these symbols, as usual, representing the sums of the reciprocals of the radii for the crown and flint respectively. Or the relations between $\frac{1}{\rho_1}$ and $\frac{1}{\rho_2}$ may be calculated for proper colour correction with tolerably near approximation by the ordinary methods, such, for instance, as conditioning that the rays F and G shall be brought to the same focus, trusting to a subsequent slight modification of one of the surfaces for neutralising the slight over-correction which would follow.

But, assuming that the relations between $\frac{1}{\rho_1}$ and $\frac{1}{\rho_2}$ are very approximately known, we have next to consider what conditions are necessary to secure the largest possible field of view. Objectives can and do vary enormously in this respect. I could show you an objective of 4½ inches aperture, with a convexo-concave crown, which yields an image which falls off so rapidly on leaving the optic axis, that even when using a Huygenian eyepiece of rather a high power, the bad definition towards the edge of the visual field is most striking. And, as necessarily follows, such an objective is keenly sensitive to the slightest tilt; if it is thrown "out of square" to even the slightest degree, good definition is quite destroyed.

The degree to which this imperfection is present is not a matter of curvature of the image, for it can be demonstrated that all objectives, of whatever form, yield images whose degree of curvature varies only between very narrow limits indeed, accord-

1893MNRAS..53..359T

ing to the sorts of glasses employed, as I shall presently show; nor is it a matter of astigmatism either, for it can be shown that the amount of marginal astigmatism (for a standard degree of obliquity) is a constant quantity for a given focal length, and totally independent of either the curves or the sorts of glasses employed.

It may be stated broadly that the characteristic of the objective which determines whether its field of fair definition shall be small and confined to a few minutes of arc only, or extended to two degrees square or so, as required in the photographic telescope, is the degree in which it is free from the defect of "coma," or what may also be called eccentric or *unsymmetrical oblique refraction.* In Plate 7, fig. 1, L represents an achromatic and aplanatic lens, by which the oblique parallel rays (from a star) D, B, C, *b*, and *d* are refracted to a focus at *f*. Let it be supposed that the curves of L are bulged outwards somewhat towards the left, then the form of the focus at *f* will be very peculiar. Let C be that ray which passes approximately through the centre of the objective and meets with no deviation. Let B and *b* be a pair of rays at equal distances on each side of C, and D and *d* another pair of rays also symmetrical with respect to C, but still wider apart, edge rays in fact. Confining our attention to rays in primary planes (the plane of the paper being the principal primary plane), it can be shown that any pairs of rays, such as B and *b*, D and *d*, situated at equal distances from, but on opposite sides of C, come to a focus upon the primary focal curve P - - *f*, but while the ultimate focus for rays close about C is at the point C on the curve P - - *f*, the rays B and *b* focus at a point B + *b*, which is displaced laterally towards the optic axis; while the extreme pair of rays D and *d* focus at the point D + *d*, which is displaced towards the optic axis by a still larger amount. The ordinates of the curve P - - *f*, measured from the axis P - - P (the principal focal plane), are represented by

$$-\mathrm{F}\,\frac{\tan^2\phi}{2}\left(\frac{3\mu+1}{\mu}\right)$$

where F is the principal focal length, $\phi$ is the angle of obliquity, and $\mu$ is (for shortness) a sort of mean between the indices of refraction for the two or more lenses of which the O.G is composed. The curve P - - *f* may be described approximately as a circle tangent to P–P, and of a radius equal to

$$\mathrm{F}\,\frac{\mu}{3\mu+1}.$$

Thus the radius of this curve, P - - *f* will be found to be generally about $\frac{3}{11}$ of the focal length, while in secondary planes the focal curve is of a radius equal to

$$\mathrm{F}\,\frac{\mu}{\mu+1},$$

1893MNRAS..53..359T

or generally about $\frac{3}{5}$ of the focal length. The mean between these values or

$$F \frac{\mu}{2\mu + 1},$$

therefore represents the radius of curvature of the curve or locus *of circles of least confusion*, or the curve along which the most distinct image is formed. Thus the most distinct image is formed on a curve having a radius of curvature of $\frac{3}{8}$ of the focal length, generally speaking.

Returning again to the lateral displacement along the primary focal curve P - - *f*, to which the foci of the symmetrical pairs of rays B and *b*, D and *d*, are subject, it will be seen readily from fig. 1 that the consequence of this peculiar arrangement is that the foci for any *eccentric* pairs of rays, such as D and B, *b* and *d*, B and C, C and *d*, &c., are thrown altogether off the focal curve P - - *f*. The pair D and B focus at the point D + B, far short of the curve P - - *f*, while such an eccentric pair of rays as *b* and *d*, lying on the other side of C, have their focus or point of intersection quite beyond the curve P - - *f*, at a point (beyond the edge of the diagram) which may be actually beyond the principal focal plane P - - P. I have marked most of the intersection points for any two rays by corresponding couples of letters. Of course the diagram is exaggerated for the sake of clearness.

The effect of placing a circular aperture or diaphragm *behind* the lens L becomes evident on inspection. If it is placed as shown in fig. 1, then only the *eccentric* pair of rays C and *b* are allowed to pass, and these intersect or have their focus at the point C + *b*, just about on the principal focal plane P - - P; thus the diaphragm or stop *may* render the field of view perfectly flat for rays in primary planes. But if the stop were placed in front of L, it is evident that the effect would be to render the image more curved than before.

Now, if a piece of ground glass is placed across the focus at *f*, it will be found, if the source of light is bright enough, that there is no distinct image such as a point, line, or circle of light, but a curious balloon-shaped penumbra (sometimes called a "coma," if not too violent), which I have endeavoured to represent in fig. 1 *a*. *r* is the smaller and brighter end, and is where the most intense concentration of rays occurs, it being the place about which the ultimate centre ray and those close about it focus at C. The larger end, *s*, is much less bright and more diffused, and corresponds to where the more extreme rays, such as D and *d*, come to focus.

Thus in the case represented in fig. 1 the greater bulk of the coma or penumbra falls *towards* the optic axis of the lens, and the brighter end away from it. Now in fig. 2 is shown another form of lens whose curves are bulged towards the right hand. Here the manner in which symmetrical and eccentric pairs of oblique rays respectively are refracted by the lens is exactly opposite in character to that exhibited in fig. 1; $C_1$ being the ultimate focus for the central ray $C_1 - - C_1$, and the focal points $B_1 + b_1$, $D_1 + d_1$, for symmetrical pairs of rays $B_1$ and $b_1$ and $D_1$ and $d_1$ being displaced along the primary focal curve $P - - f_1$ in the opposite direction to that prevailing in fig. 1. In the case of fig. 2, therefore, the "coma" or penumbra, fig. 2 *a*, is opposite in character to fig. 1 *a*, its more diffused bulk being thrown *away from* the optic axis $A_1 - - A_1$, and inspection of fig. 2 makes it clear that in order to flatten the field of view a diaphragm or stop should be placed in front of the lens, and not behind it. The behaviour of these two lenses is thus opposite in character; they both give what I may conveniently call unsymmetrical oblique refraction, but of opposite tendencies; therefore it will easily be seen that a lens intermediate in form between that shown in fig. 1 and that shown in fig. 2 should be possible, which would be free from this unsymmetrical oblique refraction. This is actually the fact. Fig. 3 illustrates the action of such a lens.

In figs. 1 and 2 we get a focus for oblique rays in which are present the inevitable faults of astigmatism and curvature of field, with the additional fault of "coma" superimposed; while in fig 3 we get astigmatism and curvature of field only, without the further blemish of "coma," and consequently if we are using a lens at open aperture, as is done in the case of telescope objectives for photographic purposes, then the best possible image off the axis is secured when such objective is designed to be free from "coma." On inspecting fig. 3, since it is seen that all pairs of rays whatsoever, whether symmetrical or eccentric, intersect or come to focus (in primary planes) at one and the same point $f_2$, *it follows that no diaphragm or stop, whether placed in front of or behind the lens, can exert any effect whatever in the way of either flattening or rounding the field of view.* Therefore if an objective is required which will give the best possible definition off the optic axis when used at open aperture, then its curves must be so calculated that its field of view cannot be either flattened or rendered more curved by the presence of any stop, either in front of or behind the objective; or, in other words, its diaphragm corrections must be eliminated or reduced to zero.

A formula is therefore required which will render this calculation possible. Rather more than a year ago I undertook a mathematical investigation of the form of an oblique pencil of rays after refraction through one or more lenses, both when such lenses were used at open aperture, and also when armed with stops or diaphragms, either in front or behind them, partly on the lines laid down by Coddington in his *System of Optics*, but in my own investigations dealing with large apertures and taking fully into account the effects of spherical aberration upon oblique rays at each surface. I undertook this on independent lines, partly

1893MNRAS..53..359T

because I could not follow Coddington throughout his processes, for he does not always clearly explain himself, as students of his work often realise. I cannot give within reasonable space the processes* by which I arrived at the general formula which I here venture to present, and which I have found to be in exact agreement with practice, provided that the thicknesses of the lenses are not disproportionately large. The formula represents a correction to the *reciprocal* of the focal length, and I will write it in a form suited to an objective of two lenses :

$\mu$ = refractive index of 1st lens
M = " of 2nd lens
$f_1$ = principal focal length of 1st lens
$f_2$ = " of 2nd lens

$\alpha_1$ (as in *Coddington's Optics*) is the characteristic of the relative state of divergence or convergence of the axial rays entering the 1st lens; thus if + or $-u$ is the distance (measured axially) from which rays are diverging, or to which they are converging respectively into 1st lens, then

$$\frac{1+\alpha_1}{2f_1}=\frac{1}{u}$$

as the case may be; while

$$\frac{1-\alpha_1}{2f_1}=\frac{1}{v}$$

or the reciprocal of the corresponding conjugate focus, with its sign assessed consistently with that of $\frac{1}{u}$.

$\alpha_2$ expresses the same for the 2nd lens as $\alpha_1$ does for the 1st lens.

$x_1$ is the characteristic of the *shape* of the 1st lens, in the same sense as in Coddington's notation.

Thus, let

$$\frac{1}{\rho_1}=\frac{1}{r_1}+\frac{1}{s_1}$$

or the sum of the reciprocals of the two radii, which are of the same (+) sign if turned in opposite ways (as in a double convex or double concave lens), and of opposite signs when turned the same way (as in a positive or negative meniscus lens) ; then

$$\frac{1+x_1}{2\rho_1}=\frac{1}{r_1}\text{ and }\frac{1-x_1}{2\rho_1}=\frac{1}{s_1}.$$

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* The formulæ previously given are deduced from these processes.

1893MNRAS..53..359T

or, as Coddington expresses it,

$$\frac{1+x_1}{2(\mu-1)f_1}=\frac{1}{r_1} \text{ and } \frac{1-x_1}{2(\mu-1)f_1}=\frac{1}{s_1}.$$

Thus, according to Coddington's beautiful device, if a lens is equi-convex, then its characteristic $x=0$; if it is convexo-plane, then $x=+1$; if plano-convex, then $x=-1$; and, it always being premised in this system that light passes through lenses from left to right, a meniscus with its convex side towards the left has its characteristic $x=$ something greater than 1; if turned round the other way, $x$ is minus quantity greater than 1, its numerical value being the same. Thus, turning a lens back to front merely changes the *sign* of its characteristic $x$.

$x_2$=the characteristic of the shape of the 2nd lens.

U=the axial distance from the 1st lens of the point from which the light is diverging.

D = the distance of the stop or diaphragm from the centre of the first lens. When D is in front of a lens (whether a positive or negative lens) then it is of + sign, but when behind a lens it is of − sign.

In the following formula it should be carefully borne in mind that the two terms U and D, wherever they occur, have their value determined in reference to the *first* lens only; for, in the formulæ extended for three, four, or more thin lenses in contact, the values of U and D, if the stop is in front (or V and −D if the stop is behind), must be determined solely in relation to that lens which lies next to the stop, the necessity for allowing for *virtual* images of the stop being obviated in this method.

$\phi$ = the angle of obliquity, or the angle included between the optic axis and that ray of the entering pencil which passes through the centre of the lens.

Premising, moreover, that the corrected focal distance for an oblique pencil is measured parallel to the optic axis, or, in other words, referred to the axis by dropping a perpendicular to the latter, and that the *axial* focal length is the standard to whose reciprocal the correction expressed by the following formula is to be applied in order to arrive at the focal length for the oblique pencil, then we have the following formula relating to the case of a double objective with crown lens placed first and with a stop or diaphragm placed axially at a distance D in front of the crown lens.

Then the correction to the reciprocal of the axial focal length is, for rays in *primary* [*] planes:

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[*] The formula for rays in secondary planes is exactly the same, except that $3\tan^2\phi$ is replaced by $\tan^2\phi$.

1893MNRAS..53..359T

$$\frac{3\tan^2\phi}{8f_1^3\mu(\mu-1)}\left[\left\{\frac{\mu+2}{\mu-1}x_1^2+4(\mu+1)\alpha_1x_1+(3\mu+2)(\mu-1)\alpha_1^2+\frac{\mu^3}{\mu-1}\right\}\right.$$

$$\left.+2f_1\left\{4\mu^2\alpha_1+2(\mu+1)(x_1-\alpha_1)\right\}\frac{\mathrm{U}-\mathrm{D}}{\mathrm{UD}}\right]\frac{\mathrm{D}^2\mathrm{U}^2}{(\mathrm{U}-\mathrm{D})^2}$$

$$-\frac{3\tan^2\phi}{8f_2^3\mathrm{M}(\mathrm{M}-1)}\left[\left\{\frac{\mathrm{M}+2}{\mathrm{M}-1}x_2^2+4(\mathrm{M}+1)\alpha_2x_2+(3\mathrm{M}+2)(\mathrm{M}-1)\alpha_2^2+\frac{\mathrm{M}^3}{\mathrm{M}-1}\right\}\right.$$

$$\left.-2f_2\left\{4\mathrm{M}^2\alpha_2+2(\mathrm{M}+1)(x_2-\alpha_2)\right\}\frac{\mathrm{U}-\mathrm{D}}{\mathrm{UD}}\right]\frac{\mathrm{U}^2\mathrm{D}^2}{(\mathrm{U}-\mathrm{D})^2}.$$

The first two lines express the diaphragm corrections for the first or crown lens in the way of alteration in the curvature of its image, dependent upon the presence of the stop in front; while the last two lines express the corrections for the second or negative lens due to the presence of the same stop in front. Now we have already concluded that the form of objective which will give the largest possible field when used at open aperture, or yield the best possible lateral image, is a form which cannot in any way have its field of view modified by the presence of a stop placed at any distance either in front of or behind it, an objective, in fact, *whose curves are such that the formulæ expressing its diaphragm corrections equate to* 0.

On inspecting the above formulæ, students of optics familiar with Coddington's work* will at once recognise in the first and third lines the formulæ which express the spherical aberration of the objective, only that $y^2$ is replaced by the common factor

$$\left(\frac{\mathrm{DU}}{\mathrm{U}-\mathrm{D}}\right)^2 3\tan^2\phi.$$

These two lines multiplied by the above factor then express those diaphragm corrections or modifications of the curvature of field which are dependent upon the presence of spherical aberration in the objective. But since a properly constructed

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* Those who have at all followed Coddington in his investigation of the form of a pencil of rays after refraction through a lens eccentrically and obliquely, will be aware that the results he arrives at are decidedly confusing, diaphragm corrections due to the spherical aberration of the lens and diaphragm corrections depending upon the *form* of the lens (on which latter the amount of "coma" is dependent) being altogether mixed up. His results are only useful when applied to pencils of very small aperture, and there is little doubt that it was the confusion I have just alluded to, which prevented him realising the advantages which one form of lens may have over another when used at open aperture, and prompted him to write (See his *System of Optics*, page 159): "The next question that offers itself is the advantage to be derived from a combination of lenses when a pencil passes through it centrically but obliquely. It will, however, easily be seen that as *the effects of obliquity in this case are totally independent of the form of a single lens*, so they cannot be removed or diminished by any combination." The italics are my own.

1893MNRAS..53..359T

objective is necessarily free from spherical aberration, these first and third lines of the formulæ may be neglected as being equal to zero, and we get, after crossing out common factors,

$$\frac{1}{f_1^2\mu(\mu-1)}\left\{2\mu^2 a_1+(\mu+1)\ (x_1-a_1)\right\}$$

for crown lens

$$+\frac{1}{f_2^2 M(M-1)}\left\{2M^2 a_2+(M+1)\ (x_2-a_2)\right\}$$

for negative lens.

Expressing $f_2$ by $\delta f_1$, this formula will be found to equate to 0 when

$$x_2=-x_1\frac{\delta^2 M(M-1)(\mu+1)}{\mu(\mu-1)(M+1)}-\frac{(M-1)\{\delta^2 M(2\mu+1)a_1+\mu(2M+1)a_2\}}{\mu(M+1)}.$$

This is the condition under which the objective is free from "coma," and therefore yields the best possible lateral image. As a concrete instance, let an objective of ordinary crown and flint be considered where, since the incident rays are parallel,

$$a_1=-1;\ \mu_G=1;\ \cdot5276 \text{ and } M_G=1\cdot6415,$$

and $\delta$, or the dispersive ratio for the spectrum interval F to G = 1·762, which is therefore the focal ratio, on the condition that the rays F and G are brought to the same focus. Inserting these values in the above formula, we get

$$x_2=-3\cdot88234\ x_1+\cdot66056.^*$$

This value for $x_2$ may then be inserted in the formulæ for spherical aberration, which will be found to equate to 0 when

$$x_1=-\cdot255.$$

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* There is another method of arriving at the curves for an objective free from "coma," provided that the most exact result is not required. It can be proved that if an objective is so designed that it shall be as nearly as possible aplanatic for converging and diverging rays as well as for parallel rays, then it will be very approximately free from coma when focussed on infinitely distant objects. Thus, if the formula expressing the spherical aberration of the objective is differentiated with respect to $a_1$ and $a_2$ (changes in the values of which, of course, express changes in the state of convergence or divergence of the entering rays), and the relative values of $x_1$ and $x_2$ which are necessary to equate the differential coefficient to 0 when $a_1 = -1$ are then inserted in the complete formula for spherical aberration, then the latter, on being equated to 0, will yield the proper value for $x_1$ and $x_2$. This method should and does yield a form of objective whose curves are rather more bulged towards the focus than the form obtained by the more exact method of eliminating diaphragm corrections.

1893MNRAS..53..359T

Therefore the radius of the first surface of the crown lens will be to the radius of the second surface as 5 : 3, broadly speaking. This calculation is exactly verified by practical results. In Fig. 4 I have represented an objective of this form. The inner curve of the flint is only very slightly flatter than that of the crown, and if a crown of lower dispersive power, such as Schott's boro-silicate crown (a very hard and beautifully colourless glass) were united with a dense flint, such as is used for ordinary prisms, then all the optical conditions may be satisfied consistently with the two inner curves being worked exactly to the same radius, so as to permit of cementing, if the objective is not too large for this to be practicable.

On tilting such an objective as this ever so much out of square, no trace of coma will be shown, the appearances being astigmatism pure and simple, as shown in Fig. 3 *a*, and varying in amount according to the square of the tangent of the angle of obliquity. It scarcely needs pointing out that an objective approaching this form is also highly desirable for transit instruments, whose eyepieces are often provided with a large amount of lateral travel.

*Buckingham Works, York:*
1893 *April* 8.

## *Results of Double-Star Measures with the 8-inch Equatoreal at Windsor, N. S. Wales, in 1892.* By John Tebbutt.

| Ref. No. | Star. | Observed Magnitude. | Approx. Place of Star. R. A. | Approx. Place of Star. Dec. S. | Fraction of Year. | Position Angle. | No. of Obs. | Distance. | No. of Obs. | Mag. Power. | Hour-Angles. | | Weight, 1 to 5. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | h m | ° ′ | | ° | | ″ | | | h m | h m | |
| 1 | *p* Eridani | ... | 1 36 | 56 45 | ·750 | 225·7 | 10 | ... | ... | 300 | 2 50 E | 2 27 E | ... |
| 2 | „ | ... | „ | „ | ·925 | ... | ... | 7·01 | 6 | 170 | 1 28 E | 1 17 E | 4 |
| 3 | „ | ... | „ | „ | ·925 | 226·4 | 10 | ... | ... | 230 | 1 3 E | 0 44 E | 4 |
| 4 | „ | ... | „ | „ | ·928 | 226·9 | 10 | ... | ... | 300 | 1 30 E | 1 11 E | 4 |
| 5 | *h* 4373 | 9, 9 | 10 44 | 41 0 | ·457 | ... | ... | 6·01 | 5 | 100 | 2 31 W | 2 41 W | 2 |
| 6 | α Crucis | ... | 12 21 | 62 30 | ·482 | 120·0 | 10 | 4·92 | 6 | ... | 2 17 E | 1 47 E | 3 & 2 |
| 7 | „ | ... | „ | „ | ·542 | 116·6 | 10 | 4·82 | 7 | 300 | 1 33 W | 2 3 W | 4 |
| 8 | γ Centauri | ... | 12 36 | 48 22 | ·441 | 0·0 | 10 | ... | ... | 300 | 2 22 E | 2 4 E | 4 |
| 9 | „ | 6, 6 | „ | „ | ·444 | 359·0 | 10 | ... | ... | 300 | 2 13 E | 1 50 E | 4 |
| 10 | „ | ... | „ | „ | ·446 | ... | ... | 1·46 | 10 | 170 | 2 47 E | 2 26 E | 4 |
| 11 | „ | ... | „ | „ | ·449 | ... | ... | 1·42 | 10 | 170 | 2 37 E | 2 21 E | 4 |
| 12 | „ | 4, 4 | „ | „ | ·457 | ... | ... | 1·94 | 10 | 170 | 1 1 W | 1 24 W | 3 |
| 13 | „ | 4, 4 | „ | „ | ·460 | 359·8 | 10 | 1·71 | 10 | 300 | 2 2 E | 1 27 E | 4 |
| 14 | „ | ... | „ | „ | ·482 | 357·3 | 10 | 1·69 | 10 | 300 | 1 47 E | 1 10 E | 5 |
| 15 | „ | ... | „ | „ | ·487 | 359·0 | 10 | 1·72 | 8 | 300 | 1 28 E | 1 6 E | 4 |
| 16 | „ | ... | „ | „ | ·597 | 358·0 | 10 | 1·82 | 7 | 535 | 2 0 W | 2 29 W | 4 |
| 17 | „ | ... | „ | „ | ·600 | 357·9 | 10 | 1·50 | 7 | 535 | 1 50 W | 2 15 W | 5 |
| 18 | γ Virginis | ... | 12 56 | 0 51 | ·485 | 151·7 | 10 | 5·51 | 8 | 300 | 1 31 E | 1 4 E | 4 |